Warm-up exercises

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Exercise 1 (Proposition 1.1.3, Baccelli–Blaszczyszyn 2009)**.** Let Φ be a Poisson point process on $\mathbb{R}^d$ with intensity measure Λ. Let us prove the following.

(i) If Λ has a fixed atom at $\{x_0\}$, $x_0 \in \mathbb{R}^d$, then Φ has an atom at x_0.

(ii) If Λ has a density w.r.t. the Lebesgue measure, then Φ is simple.

Exercise 2 (Theorem 1.1.4, BB09)**.** Let us show that a point process Φ is a Poisson point process if and only if there exists a locally finite measure Λ on $\mathbb{R}^d$ such that for all bounded Borel sets $A \subset \mathbb{R}^d$, $\Psi(A) \sim \text{Poi}(\Lambda(A))$.

Exercise 3 (Theorem 1.1.5, BB09)**.** Let Φ be a simple point process. Let us show that Φ is a Poisson p.p. if and only if there exists a locally finite non-atomic measure Λ such that $\forall A \subseteq \mathbb{R}^d$, $\mathbb{P}(\Phi(A) = 0) = \mathrm{e}^{-\Lambda(A)}$.

Exercise 4 (Theorem 1.1.7, BB09)**.** Suppose that Φ is a point process without fixed atoms. Let us prove that then Φ is a Poisson point process if and only if it satisfies the following two properties.

(i) Φ is simple,

(ii) Φ has the property of complete independence, i.e., for any $n \in \mathbb{N}$ and for any bounded and pairwise disjoint Borel sets $A_1, \ldots, A_n$, $\Phi(A_1), \ldots, \Phi(A_n)$ are independent random variables.