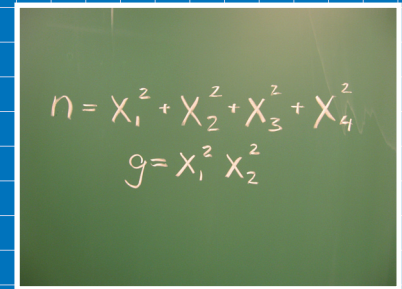**Berlin Mathematical School**

BMS Friday Colloquium

Friday, 16 July 2010, 1:45 pm

Tea before the lecture starts at 1 pm

BMS Loft, Urania
An der Urania 17, 10787 Berlin

Aner Shalev (Hebrew U Jerusalem):

"Words: From Number Theory to Finite Simple Groups"

A classical theorem of Lagrange shows that every positive integer is a sum of four squares. More generally, Waring conjectured that every positive integer is a sum of $g(k)$ k^{th} powers, and this was proved by Hilbert and others.

In recent years various group-theoretic analogues of this result were studied, in relation to Burnside problems, a question of Serre on profinite groups, and other contexts.

Here powers are replaced by general words (elements of the free group), and the aim is to present every element of a (nonabelian) finite simple group G as a short product of values of a given (non-trivial) word w.

We shall describe recent solutions to this problem, partly joint with Larsen, Tiep, Liebeck and O'Brien. We also confirm Ore's conjecture of 1951, and show that every element of a finite simple group is a product of two squares (this may regarded as a non-commutative analogue of Lagrange's theorem).

Connections with representation theory, probability and geometry will also be discussed.

$$n = X_1^2 + X_2^2 + X_3^2 + X_4^2$$
$$g = X_1^2 \, X_2^2$$