Berlin Mathematics Research Center MATH+

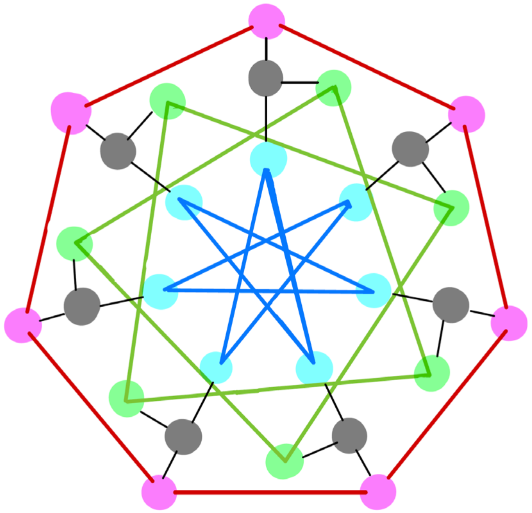
Berlin Mathematical School BMS

MATH+ Friday Colloquium

Friday, 27 June 2025 at 14:15

Urania Berlin, Old Wing (Altbau), 3rd floor, An der Urania 17

Tea & Cookies starting at 13:00



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Shigeru Mukai

(Kyoto U)

Finite simple groups and K3-like varieties

The classification of finite simple groups, comprising tens of thousands of pages and famously singling out 26 sporadic groups, is widely regarded as one of the most impressive collected efforts in the whole of mathematics. This talk is a report on Mukai's attempt to realize these very large and supremely complicated groups geometrically, as acting on very interesting algebraic varieties.

There are many instances where group theory and algebraic geometry interplay. Algebraic varieties, or complex manifolds, often have hidden symmetries leading to e.g. the theory of singularities and that of hypergeometric equations. Mukai has classified all finite groups acting symplectically on K3 surfaces (that is, on 2-dimensional Calabi-Yau varieties). Remarkably, this classification can be expressed in terms of the Mathieu group M_{23}, one of the first sporadic groups which was discovered in the 19th century.

To realize geometrically other, much larger, sporadic groups, Mukai considers actions in positive characteristic on higher-dimensional holomorphic symplectic (hyperkähler) manifolds of $K3^{[n]}$ or of O'Grady type. These are natural higher-dimensional generalization of K3 surfaces. Considering suitable such actions in characteristic $p > 0$, it is expected that three of the sporadic groups, namely the Higman-Sims group, the McLaughlin group and the third Conway group (Co_3) have such a geometric realization by reason of distance-regular graphs such as Coxeter graphs.

Shigeru Mukai is a Japanese algebraic geometer from Kyoto University. He is famous for having introduced in 1981 what came to be known as the Fourier-Mukai transform, a concept that now permeates multiple parts of modern mathematics. He has made fundamental contributions to K3 surfaces and vector bundles, to Brill-Noether theory and to the study of Fano threefolds. He has also done important work in invariant theory in connection with Hilbert's 14th problem.

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