Numerical methods for partial differential equations II

The finite element method (FEM) is a widely used numerical method for solving partial differential equations. In solid mechanics, the finite element methods are usually based on the minimization of the energy. The assumptions for the approximation to converge to the exact solution are relatively general. Together with a simple implementation, this represents significant advantage.

However, other quantities may be of great importance such as flux and stresses. If those are derived using a post-processing, the approximation is less accurate. For elasticity problems this may lead to the so-called locking effect. Therefore, mixed methods approximating displacements and stresses simultaneously with the same accuracy are of interest. Minimizing the residual of an equation and considering the other considered as constraint leads to a variational formulation with saddle point structure. The least squares method is an alternative: instead of minimizing under constraints, the residuals of the partial differential equations are minimized.