GAUGE THEORY: FROM GROUPS TO GROUPOIDS

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Abstract. In this course, we will first review the geometric (principal bundles, connections, curvature, holonomy) and analytic framework (elliptic operators, infinite dimensional Lie groups) underlying gauge theory. We will then discuss gauge groupoids built from principal bundles and equip them with connections. Gauge groupoids with connections naturally arise in various contexts, among which regularity structures. If time allows, we will discuss their higher simplicial counterparts.
To attend the lectures, please send an email to paycha@math.uni-potsdam.de in order to get the Zoom login data.

1. Geometric prerequisites

1.1. **Local versus global symmetries** [G]**.**

1.2. **Principal and associated vector bundles.**

1.3. **The gauge group as a Lie group** [Na]**.**

1.4. **Connections and curvature on principal bundles** [KN]**.**

2. The gauge group action

2.1. **The Yang-Mills functional.**

2.2. **Ellipticity and Fredholm operators.**

2.3. **The slice theorem** [W]**.**

2.4. **The moduli space of flat connections** [M]**,** [Do]**.**

3. Chern-Simons action

3.1. **Chern-Weil and Chern-Simons forms.**

3.2. **The classical Chern-Simons action in three dimensions** [De]**.**

3.3. **Holonomy and the Ambrose Singer theorem** [B]**.**

3.4. **Path integrals in Chern-Simons theory** [F]**.**

4. From principal bundles to gauge groupoids

4.1. **Groupoids and Lie algebroids: examples** [MK]**.**

4.2. **Groupoids equipped with direct connections** [Ku, KT, ABFP]**.**

4.3. **Jet prolongation of gauge groupoids** [KMS]**,** [Ko1, Ko2, Ko3]**,** [ABFP]**.**

4.4. **Higher gauge transformations.**

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