Syllabus "Computational Finance"

In mathematical finance, the price of derivatives such as options are represented as expectations of random variables, obtained from stochastic models of the underlying. Usually, explicit formulas for the prices are not available, i.e., explicit calculations of those expectations are not possible. Therefore, numerical approximation plays an important role in the finance industry. There are three general approaches to the numerical calculation of expected values.

1. By the law of large numbers, sample averages converge to the expected value of a random variable if the sample size goes to infinity. This observation leads to Monte-Carlo simulation and its variants like Quasi Monte-Carlo simulation. It requires a method to simulate from the distribution of the underlying random variable. While exact simulation is usually not possible, approximate simulation methods (e.g, Euler approximations of SDEs) are widely available. Therefore, Monte-Carlo simulation is a very general approach to approximate option prices.
2. If the underlying model is a Markovian model (e.g., given by an SDE), the option price satisfies a PDE, the Kolmogorov-backward equation. Therefore, one can compute the option price by solving the PDE numerically using the finite-difference or finite-element approach. Apart from regularity and (too) exotic path-dependence, the applicability of PDE-based approximation methods is mainly limited by the dimension of the underlying ("curse of dimensionality").
3. If explicit densities are available, expectation can be written as (low-dimensional) integrals. The density, however, is usually not known explicitly, and even if it is known, direct quadrature (i.e., numerical approximation) of the integral might not lead to a competitive numerical method. However, in many important cases (e.g., Levy or affine processes), the Fourier transform of the density (corresponding to the characteristic function of the underlying random variable) is explicitly known, thus allowing calculating the option price using Fourier methods.

In this course, we present the above mentioned approaches. Additionally, we will also present an example of an approach specifically developed for the pricing of American options. More precisely, the content of the course will be a selection of the following:

- SDEs & Finance, a reminder (including Levy processes, Ito-formula, Kolmogorov backward equation)
- Pseudo random numbers (random number generation on the computer)
- Basics of Monte Carlo simulation
- Quasi Monte Carlo
- Monte-Carlo simulation of diffusion models: weak and strong approximations, order of the Euler scheme, higher order schemes)
- Monte-Carlo simulation of jump models (diffusion plus finite activity jumps; pure jumps such as VG, CGMY)
- Solving a PDE using finite differences (various finite difference schemes, in part. Crank-Nicolson)
- Option pricing with FFT (the Carr-Madan method -- but no detailed presentation of FFT itself)
- Pricing American options (a la Longstaff & Schwarz)
- Cubature on Wiener space

Prerequisites: Sound knowledge of stochastics and finance as acquired from the courses FiMa 1 + 2. We will aim to give the course in a way, that parallel attendance of FiMa 2 is possible.

Literature:
Most relevant literature for the course will be:

- Glasserman, Paul: Monte Carlo Methods in Financial Engeneering.
- Cont, Rama and Tankov, Peter: Financial modelling with jump processes.
- Wilmott, Paul: Paul Wilmott on Quantitative Finance, Vol. 2.