An introduction to Malliavin's calculus and to its applications

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Format of the lecture : two hours lecture a week without exercises
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Language of the Lecture : English

Description of the lecture

Recently Malliavin's calculus has been successfully applied in many fields of applications including Statistics or limit theorems theory. The increasing number of research papers presenting new methods and new technologies involving Malliavin's calculus indicates the interest of the community for this field.

In this Lecture we first propose to give an introduction to Malliavin's calculus itself in a Gaussian and a Poisson context. We will describe the so-called *Malliavin derivative* and its adjoint operator the Skorohod integral related to each other *via* the integration by parts formula which plays a crucial role in every application. We will emphasize that this construction is unique in the Gaussian setting whereas several gradients and several integration by parts formulae are available in the Poisson context. Indeed, the development of this theory for jump processes is still a subject of research. The second half of this Lecture consists of presenting recent applications of the material exposed above. Our first main area of application will be Statistics. More precisely, we will present a construction of Stein type estimators for the estimation of the drift of Gaussian processes. This construction will then be extended to the Poisson context. Naturally, both of these constructions will be based of Malliavin's calculus and of its integration by parts formula. To complete this illustration of the use of Malliavin's calculus in Statistics we will present infinite dimensional Bayesian estimators for Gaussian and Poisson channels expressed in terms of some Malliavin derivatives. Obviously, the two applications aforementioned constitute only examples among the many papers realized recently involving the use of Malliavin's calculus in Statistics. The second main field of applications we will describe is the use of Malliavin's calculus to provide limit theorems for non-semimartingales stochastic processes and especially for the *fractional Brownian motion* and for its two-parameter counterpart the *fractional Brownian sheet*. After having introduced the definition and the main properties of these stochastic processes we will present limit theorems for their weighted variations (which are needed for example in the study of numerical schemes for SDE driven by these processes). We will conclude this Lecture by giving an introduction to the interaction between Malliavin calculus and the Stein's method which is a method very useful both for deriving limit theorems and for Statistics.