Finite size effects: Random matrices, quantum chaos and Riemann zeros

Since the legendary 1972 encounter of H. Montgomery and F. Dyson at tea time in Princeton, the statistical correspondence of the non-trivial zeros \( \frac{1}{2} + iE \) of the Riemann zeta function with eigenvalues of high-dimensional random matrices poses a big open mathematical problem. An explanation for this mystery is conjured up as follows: if the values \( E \) formed the spectrum of a Hamiltonian operator with chaotic classical mechanics, then they would behave statistically like eigenvalues of random matrices (quantum chaos). Thanks to extensive calculation of Riemann zeros by A. Odlyzko, overwhelming numerical evidence can be found for the correspondence. In the largest data set of \( 10^9 \) zeros at height \( E = 10^{22} \), even finite size effects can be observed in the fluctuation statistics whose precise prediction was recently made possible by the numerical evaluation of operator determinants and their perturbation series (joint work with P. Forrester and A. Mays, Melbourne).

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