On monotone operators, evolution equations, and existence via discretization

The mathematical description of many processes in science and engineering leads to nonlinear time-dependent partial differential equations. A particular example is the Navier-Stokes equation describing the flow of an incompressible, viscous fluid, which is of first order in time. Equations of second order appear in elastodynamics, e.g. in the description of a vibrating membrane. One of the main goals is to prove existence of solutions to such equations. In order to do so, we may consider a numerical approximation providing a sequence of approximate solutions. Stability of the numerical scheme allows us to derive a priori bounds for these approximate solutions, which implies (weak or weak*) convergence of a subsequence. It turns out that the underlying differential operators often share a property that generalizes the notion of a monotonically increasing function. It is this property that allows us to identify the limit of a subsequence of approximate solutions as a solution to the original problem. Another interesting point is that stability of a numerical scheme like a time discretization method is nothing else than an appropriate discrete counterpart of the integration-by-parts formula.

In his talk, Emmrich will focus on a few of the above main steps and will end with a discussion of new results concerning the existence of solutions to nonlinear evolution equations of second order.

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