As was already emphasized in C. F. Gauss’ *Disquisitiones Arithmeticae*, the concept of congruences (also known as arithmetic progressions of some modulus \( q \), or the ring \( \mathbb{Z}/q\mathbb{Z} \)) is central in number theory, and it is therefore fundamental to understand spaces of functions on the ring \( \mathbb{Z}/q\mathbb{Z} \), especially when \( q \) is a prime. The first class of functions to have been studied systematically (and one of the most interesting even today) is the group of Dirichlet’s characters (the group of homomorphisms from the multiplicative group \((\mathbb{Z}/q\mathbb{Z})^*\) to \(\mathbb{C}\)); these play a crucial role in Dirichlet’s Theorem in proving that there are infinitely many primes in any admissible arithmetic progression.

During the 20th century, a rich class of functions of algebraic geometric origin has emerged: the “trace functions”. Trace functions occur as traces of certain endomorphisms acting on \( (\ell\text{-adic}) \) cohomology groups of algebraic varieties over the finite field \( \mathbb{F}_q \). They have been studied in depth by the Grothendieck school, notably by P. Deligne and N. Katz. In his lecture, Michel will describe some applications of trace functions to number theory, for instance to Y. Zhang’s proof of the existence of bounded gaps between primes.

Philippe Michel is a French mathematician who holds the chair in analytic number theory at the École Polytechnique Fédérale de Lausanne (EPFL) in Switzerland. Michel did both his PhD and habilitation at U Paris-Sud in 1995 and 1998, respectively. From 1998 onwards, he was a professor at the U Montpellier, before moving to EPFL in 2008. In 1999, Michel was awarded the Peccot-Vimont Prize, and in 2006, he was an invited speaker at the ICM in Madrid. In 2012, he became a fellow of the American Mathematical Society.