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The Borel Conjecture

A differentiable manifold has a homotopy type, a homeomorphism type, a diffeomorphism type, and often there is additional structure, like for example a Riemannian metric.

Whereas a complete classification of manifolds up to homotopy, homeomorphism or diffeomorphism is hopeless, relative questions are more tractable. Under suitable assumptions one expects rigidity phenomena, i.e., that there exists only one homeomorphism type within a given homotopy type. The most famous such instance is the Poincaré Conjecture, now a theorem: A manifold that is homotopy equivalent to a sphere is homeomorphic to a sphere.

A little less well-known is the Borel Conjecture, which in a certain sense is orthogonal to the Poincaré Conjecture: A manifold that is homotopy equivalent to an aspherical manifold is already homeomorphic to it. A manifold is called aspherical if its only non-vanishing homotopy group is the fundamental group. Every Riemannian manifold with nonpositive sectional curvature is an example of an aspherical manifold.

Astonishingly, algebraic questions need to be solved in order to make progress on the Borel Conjecture.

The talk will give an introduction to this circle of ideas suitable for a general mathematical audience and report on known results, which are mostly due to Tom Farrell, Lowell Jones, Arthur Bartels, Wolfgang Lück and Holger Reich himself.